PRESSURE-DIFFERENCES IN THE FREE AIR.

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[Translation of Abstract by J. Hann in Met. Ztschr., März 1914, 31. Jhrg., p. 143.]

If we represent by p and $p+\Delta p$ the air pressure at two places having the same level, and seek the variations of the pressure-differences with increase of altitude (dh) in the verticals of those places, we obtain the expressions

$$dp = -\rho dh,$$
$$d(p+\Delta p) = -(\rho+\Delta\rho)dh$$

whence

$$d(\Delta p) = -\Delta\rho dh.$$

Substituting for $\Delta\rho$ (variation of density with pressure and temperature) its equivalent expression, we obtain:

$$\frac{d\Delta p}{dh} = \frac{p}{RT}\left(\frac{dt}{T} - \frac{dp}{p}\right),$$

where $R = 29.3$.

The change of the pressure-differences in the verticals between the two places, for one unit in variation of altitude is represented by

$$\frac{p}{RT}\left(\frac{dt}{T} - \frac{dp}{p}\right) = 0.0341\frac{p}{T}\left(\frac{dt}{T} - \frac{dp}{p}\right).$$

Shaw would ascribe a special significance, for atmospheric dynamics, to the above difference in parentheses. The change of the wind force with altitude certainly depends on the changes of the pressure-differences between areas of high pressure and areas of low pressure.

Let us first consider the increase with altitude, of the difference of air pressure. We know that in the stratosphere it is cooler over a high pressure area ($p+dp$) than over a low pressure area; therefore dp and dt have opposite signs and consequently $\Delta t/T$ and $\Delta p/p$, respectively, increase numerically but in opposite directions, and for this reason the pressure-differences decrease very rapidly with altitude in the stratosphere itself. This is true in general at those levels which are tropospheric over a station with high pressure, and stratospheric over another with low pressure. But in the troposphere the air in an area of high pressure is warmer than in one of low pressure, and therefore in this case dt and dp have the same sign. Between the earth's surface and 9 km. T changes from about 280° to 220°, while for p the change may be assumed to be from about 760 mm. to 230 mm. Therefore the temperature decreases much more slowly than the pressure ($T'/T = 0.78$; $p'/p = 0.30$). Hence, between the earth's surface and 9 km. dp/p varies considerably while dt/T remains relatively more stable. Therefore, between 9 km. and the earth's surface the difference ($dt/T - dp/p$) may become zero at any point, and this difference may be positive in the lower portion and negative in the upper portion. Hence, in the troposphere the effect of temperature differences will be opposed to that of the pressure-differences instead of strengthening the latter as in the stratosphere.

The second factor p/T varies between $2\frac{4}{4}$ and 1, (760/273 and 230/230 at 9 km.); hence it follows that in the greater part of the troposphere the effect of the vertical changes on the difference of air pressure is undoubtedly very trifling, since dt/T and dp/p have the same sign, except sometimes at the lowest altitudes in areas of high pressure with cold at the earth's surface.

In summing up, we may say: In the stratosphere the temperature in a vertical direction is nearly uniform; in an area of high pressure the temperature is cooler than in an area of low pressure, but in the region directly under the bounding surface of the troposphere a change occurs and in the troposphere itself the high pressure area is warmer than the low pressure area except occasionally in the lowest kilometer. The effect on the differences of air pressure is: In the stratosphere even to the lowest limit the pressure-difference increases downward rapidly to a maximum; from this level up to about 1 km. from the surface of the earth through an interval of 8 km. the variations are capricious and with opposite signs and the total effect comparatively small.

Cave has discovered that at the very beginning of the stratosphere the wind force reaches a maximum, and that in this layer itself a more or less rapid decrease in wind force takes place.

Shaw has further computed that in the stratosphere the difference of air pressure diminishes more rapidly than the pressure, somewhat in the ratio of 24/16 or 3/2. The temperature is constant, and consequently the force of the wind diminishes at the rate of 9 per cent for each kilometer of altitude.

According to Shaw the above stated ratios of pressure and temperature in the stratosphere determine its predominating influence on the dynamics of the surface layers, although it constitutes only one-fourth of the atmosphere. In the stratosphere pressure-differences form with comparative rapidity whereas in the troposphere the changes are very capricious and contradictory, for which reason the influence of the layers between 1 and 9 km. is comparatively insignificant.